Simultaneous Equations

1.

Find the coordinates of the points of intersection of the given straight line and curve in each case.

a $y = x + 2$
$y = x^2 - 4$

b $y = 4x + 11$
$y = x^2 + 3x - 1$

c $y = 2x - 1$
$y = 2x^2 + 3x - 7$

2.

The line $y = 5 - x$ intersects the curve $y = x^2 - 3x + 2$ at the points P and Q.

Find the length PQ in the form $k\sqrt{2}$.

3.

Solve the simultaneous equations

$$3^{x-1} = 9^{2y}$$
$$8^{x-2} = 4^{1+y}$$

4.

Use an algebraic method to show that the graphs

$$y = 1 - x \quad \text{and} \quad y = x^2 - 6x + 10,$$

do **not** intersect.

5.

The line straight L and the curve C have respective equations

$$L:\ 2y = 7x + 10,$$

$$C:\ y = x(6 - x).$$

a) Show that L and C do not intersect.

b) Find the coordinates of the maximum point of C.

c) Sketch on the same diagram the graph of L and the graph of C, showing clearly the coordinates of any points where each of the graphs meet the coordinate axes.

6.
The curve C has equation

$$y = 4x^2 - 7x + 11.$$

The straight line L has equation

$$y = 5x + k,$$

where k is a constant.

Given that C and L intersect at two distinct points, show that $k > 2$.

7.
The straight line L has equation

$$y = kx - 9,$$

where k is a constant.

The curve C has equation

$$y = 3(x+1)^2.$$

It is further given that L is a tangent to C at the point P.

Determine the possible coordinates of P.

8.
The straight line with equation

$$y = 2x + k,$$

where k is constant, is a tangent to the curve with equation

$$y = x^2 - 8x + 1.$$

By **using the discriminant** of a suitable quadratic, determine the value of the constant k and hence find the point of contact between the tangent and the curve.

Simultaneous Equations Solutions

1.

a $x+2=x^2-4$

$x^2-x-6=0$

$(x+2)(x-3)=0$

$x=-2$ or 3

$\therefore$ $(-2, 0)$ and $(3, 5)$

b $4x+11=x^2+3x-1$

$x^2-x-12=0$

$(x+3)(x-4)=0$

$x=-3$ or 4

$\therefore$ $(-3, -1)$ and $(4, 27)$

c $2x-1=2x^2+3x-7$

$2x^2+x-6=0$

$(2x-3)(x+2)=0$

$x=-2$ or $\frac{3}{2}$

$\therefore$ $(-2, -5)$ and $(\frac{3}{2}, 2)$

2.

$5-x=x^2-3x+2$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$x=-1$ or 3

P and Q are the points $(-1, 6)$ and $(3, 2)$

$PQ^2=(3+1)^2+(2-6)^2$

$PQ=\sqrt{32}=4\sqrt{2}$

3.

$3^{x-1}=(3^2)^{2y}$ $\quad\therefore\ x-1=4y$

$(2^3)^{x-2}=(2^2)^{1+y}$ $\quad\therefore\ 3x-6=2+2y$

$6x-16=4y$

$\Rightarrow\quad 6x-16=x-1$

$x=3$

$\therefore\quad x=3, y=\frac{1}{2}$

4.

$\left.\begin{array}{l} y=1-x \\ y=x^2-6x+10 \end{array}\right\} \Rightarrow x^2-6x+10=1-x$

$\Rightarrow x^2-5x+9=0$

$b^2-4ac = (-5)^2-4\times1\times9 = 25-36 = -11 < 0$

NO REAL SOLUTIONS

NO INTERSECTIONS BETWEEN THE GRAPHS

5.

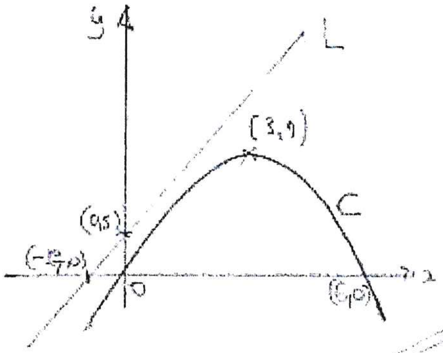
(a) $\left.\begin{array}{l} y = x(6-x) \\ 2y = 7x+10 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 2y = 2x(6-x) \\ 2y = 7x+10 \end{array}\right\} \Rightarrow$

$$2x(6-x) = 7x+10$$
$$12x - 2x^2 = 7x+10$$
$$0 = 2x^2 - 5x + 10$$

$$b^2 - 4ac = (-5)^2 - 4\times2\times10$$
$$= 25 - 80 = -55 < 0$$

HENCE NO SOLUTIONS TO THE QUADRATIC, SO L & C DO NOT INTERSECT

(b)



- $2y = 7x+10$ $\quad x=0 \quad y=5$
  $\quad y=0 \quad x=-\frac{10}{7}$
- $y = x(6-x)$ $\quad \frown$
  $y=0 \quad x = 0$ or 6
  $x=3$ (LINE OF SYMMETRY) $\Big\}$ it MAX
  $y=9$ $\quad (3,9)$

6.

$\left.\begin{array}{l} y = 4x^2 - 7x + 11 \\ y = 5x + k \end{array}\right\}$
$\Rightarrow 4x^2 - 7x + 11 = 5x + k$
$\Rightarrow 4x^2 - 7x - 5x + 11 - k = 0$
$\Rightarrow 4x^2 - 12x + 11 - k = 0$

TWO INTERSECTIONS $b^2 - 4ac > 0$

$\Rightarrow (-12)^2 - 4\times4\times(11-k) > 0$
$\Rightarrow 144 - 16(11-k) > 0$ (DIVIDE THROUGH BY 16)
$\Rightarrow 9 - (11-k) > 0$
$\Rightarrow -2 + k > 0$
$\Rightarrow k > 2$

AS REQUIRED

7.

$C: y = 3(x+1)^2$

$L: y = kx-9$

$\Rightarrow 3(x+1)^2 = kx-9$

$\Rightarrow 3x^2+6x+3 = kx-9$

$\Rightarrow \boxed{3x^2+(6-k)x+12=0}$ (*)

$\Rightarrow$ TANGENT $\Rightarrow$ REPEATED ROOTS

$\Rightarrow b^2-4ac=0$

$\Rightarrow (6-k)^2-4\times3\times12=0$

$\Rightarrow k^2-12k+36-144=0$

$\Rightarrow k^2-12k-108=0$

$\Rightarrow (k+6)(k-18)=0$

$k = -6$ or 18

- If $k=-6$

(*) $\Rightarrow 3x^2+12x+12=0$

$\Rightarrow x^2+4x+4=0$

$\Rightarrow (x+2)^2=0$

$x=-2$

$\downarrow$

$y=3(x+1)^2$

$y=3$

$\therefore P(-2,3)$

- If $k=18$

(*) $\Rightarrow 3x^2-12x+12=0$

$\Rightarrow x^2-4x+4=0$

$\Rightarrow (x-2)^2=0$

$x=2$

$\downarrow$

$y=3(x+1)^2$

$y=27$

$P(2,27)$

8.

$$\left.\begin{array}{l} y = 2x + k \\ y = x^2 - 8x + 1 \end{array}\right\} \Rightarrow x^2 - 8x + 1 = 2x + k$$

$$\Rightarrow x^2 - 10x + 1 - k = 0$$

$$\Rightarrow \boxed{x^2 - 10x + (1-k) = 0}$$

FOR A TANGENT THIS QUADRATIC MUST PRODUCE REPEATED ROOTS

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (-10)^2 - 4 \times 1 \times (1-k) = 0$$

$$\Rightarrow 100 - 4(1-k) = 0$$

$$\Rightarrow 100 - 4 + 4k = 0$$

$$\Rightarrow 4k = -96$$

$$\Rightarrow k = -24$$

USING $k = -24$ INTO $x^2 - 10x + (1-k) = 0$

$$x^2 - 10x + 25 = 0$$

$$(x-5)^2 = 0$$

$$\boxed{x = 5}$$

& USING $y = 2x - 24$

$$\boxed{y = -14}$$

$(5, -14)$